INTERNATIONAL UNIVERSITY OF SARAJEVO

SPRING 2016.

MATH 205 NUMERICAL ANALYSIS

MID-TERM EXAM

STUDENT NAME: _____

DATE: _____

Instructions:

• Examination time: **110 min**.

• Print your **name** and **student ID number** in the space provided above.

• This examination is **closed book** and **closed notes**.

• There are 4 questions. The points for each question are given in brackets, next to the question title. The overall maximum score is 100. **This mid-term weighs 20% of your final grade.**

• Answer each question in the space provided. If you need to continue an answer onto the back of the sheet, clearly indicate that and label the continuation with the question number.

QUESTION	1	2	3	4
POINTS	/20	/30	/36	/14
			TOTAL	/100

1. Mark the correct statement(s) (20 %)

1.1 Newton-Raphson method may fail when:

a) f(x) is negative
b) f'(x) is large
c) f'(x) is zero
d) The method never fails!

1.2 The number resulting from 0.01850x103 has _____ significant digits

a) 3 b) 4
c) 5 d) 6

1.3 The reason for using numerical methods is

a) one cannot always find exact solutions
b) numerical methods are more precise than analytical methods
c) one can easily experiment with model parameters
d) a modern way of doing engineering

1.4 *f(x)* is a continuous function. $f(a) \cdot f(b) > 0$ for two real numbers *a* and *b*. Then,

a) At least one root of *f(x)* is in interval [a, b]
b) No root of *f(x)* lies in interval [a, b]
c) Either no root or an even number of roots lie in [a, b]
d) We must know whether the function is differentiable before stating something about number and existence of roots

2. General (30%)

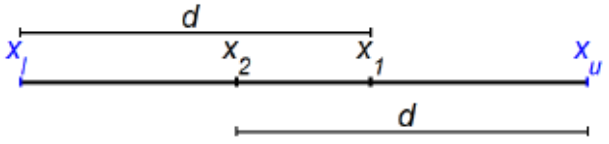
2.1 What are round-off and truncation errors? Explain in your words and possibly with some examples! **(6)**

2.2 Write the formula for finding $\ln(\mathbf{x})$ where x is a real positive number, by Newton-Raphson's method. Do not find any number, just write how to calculate the next iterate. **(6)**

2.3 How many solutions does the following system of linear equations have? Explain! **(6)**

$5x + 5y - 3z = 2$
$-x + 3y + z = 12$
$3z - y + 2x = 6$

2.4 In each iteration of the golden-section search method, there are four points involved, namely: two outer points (x_l and x_u) and two inner points x_1 and x_2. Explain why is it such a good idea to place the two inner points by a factor (ϕ-1) away from the interval edge, that is, d=(ϕ-1) (x_u-x_l) **(6)**



2.5 What does it mean if we say the function *f(x)* is **unimodal** in the interval a<x<b **(6)**

3. Roots and optima (36%)

3.1 Using regula falsi method, find the root of the function f(x) within the region 0<x<2. Do at least two iterations! **(16)**

$$f(x) = e^{x/2} - 2x$$

Number of iteration n	Interval with root
0	0 , 2
1	,
2	,
3	,

3.2 Using the Golden-section search method, find the three **positive** numbers *x*, *y*, and *z* that satisfy: *x + y = 5* and *x + z = 8* and whose product x · y · z is as large as possible!
HINT: Express y and z in terms of x and formulate an equation for optimization.
NOTE that you need to **maximize** the function, not minimize… **(20)**

f(x)=

The interval of possible *x* that results in positive *y* and *z* is from 0 to ___?
Use this interval as initial guess and ϕ=1.618. Do at least two iterations!

Iteration	x_{Lower}	x_2	x_1	x_{Upper}	$f(x_2)$	$f(x_1)$
0	0					
1						
2						
3						

x = _____ y = _____ z = _____

4. Matlab (14%)

4.1 How would you find roots of the function in question 3.1 using MATLAB? **(7)**

4.2 How would you maximize the function in question 3.2 using MATLAB? **(7)**