INTERNATIONAL UNIVERSITY OF SARAJEVO

SPRING 2019.

MATH 205 NUMERICAL ANALYSIS
FINAL EXAM

STUDENT NAME & ID: ____________________________

DATE: ____________________________

Instructions:

- Examination time: **120 min**.
- Write your **name** and **student ID number** in the space provided above.
- This examination is **closed** book and **closed** notes. You may **use a calculator** to perform the calculations required.
- There are 3 questions. The points for each question are given in brackets, next to the question title. The overall maximum score is 100. **This exam weighs 40% of your final grade.**
- Answer each question in the space provided. If you need to continue an answer onto the back of the sheet, clearly indicate that and label the continuation with the question number.

QUESTION	1	2	3
POINTS	/20	/40	/40
		TOTAL	/100

1. Answer and <u>justify</u> (20 %)

1.1 Which of the following datasets can be interpolated by a 5^{th} degree polynomial? **(5)**

a)

x	0	1	1	3	4
f(x)	0.1	1.5	0.21	3.2	1.4

b)

x	0	1	2	3	4
f(x)	0.1	1.5	0.21	3.2	1.4

c)

x	0	1	2	3	
f(x)	0.1	1.5	0.21	3.2	

d)

x	0	1	2	3	5
f(x)	0.1	1.5	0.21	3.2	1.4

1.2 List conditions, that is, characteristics of a function, which are necessary for bisection method to find its root! **(5)**

1.3 Error for the composite trapezoidal rule of integration is proportional to the: **(5)**

a) First derivative of the integrated function
b) Second derivative of the integrated function
c) Third derivative of the integrated function
d) The error decreases if the function is a polynomial

1.4 The function f(x) is evaluated at $f(1) = 2$, $f(2) = 3$ and $f(3) = 3$ where $f(x)$ is a **third order polynomial**. Imagine the integral

$I = \int_1^3 f(x)dx$ is calculated by Simpson's 1/3 formula. What happens with the truncation error if the step-size is halved? **(5)**

2. General (40%)

2.1 Using **Newton-Raphson's** method, find the value of *x* for the equation $x \cdot \ln(x) = 1.2$ **correct five decimal places**. Your initial guess can be x_0=3. **(20)**

2.2 Write down the **composite** Simpson's 1/3rd formula and compute the integral **(12)**

$I = \int_{-4}^{4} f(x)dx$ if the domain is split in **eight** equal intervals. Find I if $f(x) = e^{-x^2}$ You may use the fact that *f(x)* is even function in this case.

2.3 Find the polynomial which takes the following values using Lagrange formula **(8)**

x	*0*	*1*	*2*
f(x)	*1*	*2*	*1*

3. Caclulations and MATLAB (40%)

3.1 A dataset is given below.

X:	2	5	7	10	12
Y:	18	180	448	1210	2028

```
x=[2 5 7 10 12];
y=[18 180 448 1210 2028];
```

a) How can you estimate the value of Y at X=6 using full polynomial interpolation? Write a MATLAB code (no more than two lines) that solves the problem! **(8)**

b) How can you estimate the value of Y at X=6 using piecewise polynomial interpolation? Write a MATLAB code (no more than two lines) that solves the problem! **(8)**

c) Use any of the polynomials from a) or b) to find the definite integral of Y for 2<X<12. Write a MATLAB code (no more than two lines) that solves the problem! **(6)**

d) Use FDM and BDM to estimate the **derivative of Y** at **X=7** (use data given in table above)! Do not write MATLAB code, calculate the result using calculator. **(6)**

3.2 What does the following MATLAB code do: **(5)**

```
fminbnd(@(xx) quad(@(x) cos(x), -xx, xx), -5, 2)
```

3.3 Consider the differential equation below! How many coupled first-order differential equations can describe the same equation? State the equations! **(7)**

$$\frac{d^3 y(t)}{dt^3} + a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = 0$$

APPENDIX – USEFUL FORMULAS AND MATLAB COMMANDS

Chain rule differentiation: if ***F*(*x*) = *f*(*g*(*x*)) than *F'*(*x*) = *f'*(*g*(*x*)) *g'*(*x*)**

Product rule for differentiation: if ***F(x) = f(x) g(x)*** **than** ***F'(x) = f'(x) g(x) + f(x) g'(x)***

Segments (n)	Points	Name	Formula	Truncation Error
1	2	Trapezoidal rule	$(b-a)\dfrac{f(x_0)+f(x_1)}{2}$	$-(1/12)h^3 f''(\xi)$
2	3	Simpson's 1/3 rule	$(b-a)\dfrac{f(x_0)+4f(x_1)+f(x_2)}{6}$	$-(1/90)h^5 f^{(4)}(\xi)$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f(x) = \sum_{k=1}^{N} \left(\prod_{\substack{j=1 \\ j \neq k}}^{N} \frac{x - x_j}{x_k - x_j} \right) y_k$$

$$I = (b-a)\frac{f(x_0) + 4\sum_{i=1,3,5}^{n-1} f(x_i) + 2\sum_{j=2,4,6}^{n-2} f(x_j) + f(x_n)}{3n}$$

```
>> help polyfit
 POLYFIT Fit polynomial to data.
    P = POLYFIT(X,Y,N) finds the coefficients of a polynomial P(X) of
    degree N that fits the data Y best in a least-squares sense. P is a
    row vector of length N+1 containing the polynomial coefficients in
    descending powers, P(1)*X^N + P(2)*X^(N-1) +...+ P(N)*X + P(N+1).

>> help spline
 SPLINE Cubic spline data interpolation.
    PP = SPLINE(X,Y) provides the piecewise polynomial form of the
    cubic spline interpolant to the data values Y at the data sites X,
    for use with the evaluator PPVAL and the spline utility UNMKPP.
    X must be a vector.

>> help quad
 QUAD   Numerically evaluate integral, adaptive Simpson quadrature.
    Q = QUAD(FUN,A,B) tries to approximate the integral of scalar-valued
    function FUN from A to B to within an error of 1.e-6 using recursive
    adaptive Simpson quadrature. FUN is a function handle. The function
    Y=FUN(X) should accept a vector argument X and return a vector result
    Y, the integrand evaluated at each element of X.

>> help fminbnd
 FMINBND Single-variable bounded nonlinear function minimization.
    X = FMINBND(FUN,x1,x2) attempts to find  a local minimizer X of the function
    FUN in the interval x1 < X < x2.  FUN is a function handle.  FUN accepts
    scalar input X and returns a scalar function value F evaluated at X.

>> help ppval
 ppval  Evaluate piecewise polynomial.
    V = ppval(PP,XX) returns the value, at the entries of XX, of the
    piecewise polynomial f contained in PP, as constructed by PCHIP, SPLINE,
    INTERP1, or the spline utility MKPP.

>> help polyval
 polyval Evaluate polynomial.
    Y = polyval(P,X) returns the value of a polynomial P evaluated at X. P
    is a vector of length N+1 whose elements are the coefficients of the
    polynomial in descending powers.
```