International University of Sarajevo

SPRING 2016.

MATH 205 NUMERICAL ANALYSIS

FINAL EXAM

STUDENT NAME: ______________________________

DATE: ______________________________

Instructions:

- Examination time: **120 min**.
- Print your **name** and **student ID number** in the space provided above.
- This examination is **closed book** and **closed notes**.
- There are 3 questions. The points for each question are given in brackets, next to the question title. The overall maximum score is 100. **This exam weighs 40% of your final grade.**
- Answer each question in the space provided. If you need to continue an answer onto the back of the sheet, clearly indicate that and label the continuation with the question number.

QUESTION	1	2	3
POINTS	/20	/40	/40
		TOTAL	/100

1. Mark the correct statement(s) (20 %)

1.1 To be sure to find the zero in interval $a<x<b$, of a function *f(x)* using bisection method, the following must be satisfied:

a) *f(x)* is continuous in interval $a<x<b$

b) *f'(x)* is continuous in interval $a<x<b$

c) the zero must have odd multiplicity

d) *f(x)* is differentiable in interval $a<x<b$

1.2 Which of the following methods usually gives smaller error in finding derivative (differentiation)

a) Forward difference method

b) Backward difference method

c) Central difference method

d) They all give the same error

1.3 Error for the composite trapezoidal rule of integration is proportional to the:

a) First derivative of the integrated function

b) Second derivative of the integrated function

c) Third derivative of the integrated function

d) The error decreases if the function is a polynomial

1.4 Searching maxima and minima of functions of one variable, in Matlab we do with the command

a) `golden_sec(fun, x1, x2)`

b) `fminbnd(fun, x1, x2)`

c) `fgoldsearch(fun, x1, x2)`

d) We must develop such command

2. General (40%)

2.1 Using Newton-Raphsons method, find the root of the function *f(x)* that exists within the region $-2<x<0$. Do at least two iterations! Why is it not smart to have -1 as initial guess? Use $x_0=-1.8$ **(15)**

$$f(x)=\ln(x+2)+\frac{x^2}{2}$$

Number of iteration	Guess	Error
0	**-1.8**	**--**
1		
2		
3		

2.2 Using Simpsons 1/3 integration rule and h=1, find the result of **(15)**

$$\int_{-2}^{2}\left(-x^2+4\right)dx$$

Try to use as few calculations as possible! (It is an even function!)

What is the error? Explain!

2.3 Use the central difference approximation to estimate the first derivative of the function *f(x)=cosh(x)* at x=2 and h=0.1. **(10)**

NOTE: *cosh*(1.9)=3.4177, *cosh*(2)=3.7622, *cosh*(2.1)=4.1443

What are the absolute and relative errors of the result?

3. Matlab (40%)

3.1 You have a dataset of 10 measurements of speed of wind in the file `data.txt` that contains two columns – time and the speed of wind at that time.

a) Write a Matlab program that imports the data and interpolates a full polynomial to the dataset! **(5)**

b) Write a Matlab program that imports the data and approximates the data by a 6^{th} degree polynomial! **(5)**

c) Write a Matlab program that imports the data and approximates the data by a cubic spline! **(5)**

3.2 You have a function *f(t)* given by **(15)**

$$\boldsymbol{f(t) = \int_0^t \sin(x^3 - 7x)\, dx}$$

Write a small program that will compute and plot the function in the interval t=0…4 with 100 points!

3.3 What differential equation is solved in the following program? What are the initial conditions? What is the time span of the solution? **(10)**

`clear all`
`close all`
`[t,Y] = ode45(@mistery,[0 1],[9 5]);`
	`function dy = mistery(t,y)`
	`dy = [y(2); 3*y(2) * t];`
	`end`

To solve the exam, you may need some of the following formulas or Matlab commands:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} + O(h)$$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h} - O(h^2)$$

$$I = (b-a)\frac{f(x_0) + 4\sum_{i=1,3,5}^{n-1} f(x_i) + 2\sum_{j=2,4,6}^{n-2} f(x_j) + f(x_n)}{3n}$$

Segments (n)	Points	Name	Formula	Truncation Error
1	2	Trapezoidal rule	$(b-a)\frac{f(x_0)+f(x_1)}{2}$	$-(1/12)h^3 f''(\xi)$
2	3	Simpson's 1/3 rule	$(b-a)\frac{f(x_0)+4f(x_1)+f(x_2)}{6}$	$-(1/90)h^5 f^{(4)}(\xi)$

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>> help polyfit
 POLYFIT Fit polynomial to data.
    P = POLYFIT(X,Y,N) finds the coefficients of a polynomial P(X) of
    degree N that fits the data Y best in a least-squares sense. P is a
    row vector of length N+1 containing the polynomial coefficients in
    descending powers, P(1)*X^N + P(2)*X^(N-1) +...+ P(N)*X + P(N+1).

>> help spline
 SPLINE Cubic spline data interpolation.
    PP = SPLINE(X,Y) provides the piecewise polynomial form of the
    cubic spline interpolant to the data values Y at the data sites X,
    for use with the evaluator PPVAL and the spline utility UNMKPP.
    X must be a vector.

>> help quad
 QUAD   Numerically evaluate integral, adaptive Simpson quadrature.
    Q = QUAD(FUN,A,B) tries to approximate the integral of scalar-valued
    function FUN from A to B to within an error of 1.e-6 using recursive
    adaptive Simpson quadrature. FUN is a function handle. The function
    Y=FUN(X) should accept a vector argument X and return a vector result
    Y, the integrand evaluated at each element of X.

>> help fminbnd
 FMINBND Single-variable bounded nonlinear function minimization.
    X = FMINBND(FUN,x1,x2) attempts to find  a local minimizer X of the function
    FUN in the interval x1 < X < x2.  FUN is a function handle.  FUN accepts
    scalar input X and returns a scalar function value F evaluated at X.
```