INTERNATIONAL UNIVERSITY OF SARAJEVO

CS303 Digital Design
Active Tutorial 2
14-18/10/2019

Axioms of Boolean Algebra	**Two- and Three-Variable Properties**	
1a. 0 · 0 = 0	10a. x · y = y · x	Commutative
1b. 1 + 1 = 1	10b. x + y = y + x	
2a. 1 · 1 = 1	11a. x · (y · z) = (x · y) · z	Associative
2b. 0 + 0 = 0	11b. x + (y + z) = (x + y) + z	
3a. 0 · 1 = 1 · 0 = 0	12a. x · (y + z) = x · y + x · z	Distributive
3b. 1 + 0 = 0 + 1 = 1	12b. x + y · z = (x + y) · (x + z)	
4a. If x = 0, then x' = 1	13a. x + x · y = x	Absorption
4b. If x = 1, then x' = 0	13b. x · (x + y) = x	
	14a. x · y + x · y' = x	Combining
Single-Variable Theorems	14b. (x + y) · (x + y) = x	
5a. x · 0 = 0		
5b. x + 1 = 1	15a. (x · y)'= x' + y'	**DeMorgan's theorem**
6a. x · 1 = x	15b. (x + y)' = x' · y'	
6b. x + 0 = x		
7a. x · x = x	16a. x + x' · y = x + y	
7b. x + x = x	16b. x · (x' + y) = x · y	
8a. x · x' = 0	17a. x · y + y · z + x' · z = x · y + x' · z	Consensus
8b. x + x' = 1	17b. (x + y) · (y + z) · (x' + z) = (x + y) · (x' + z)	
9. x'' = x		

1. Simplify the following Boolean expressions to a minimum number of literals:
   a) $x + 0 =$
   b) $x' \cdot 0 =$
   c) $x + x' =$
   d) $x + x =$
   e) $x + xy =$
   f) $x + x'y =$
   g) $x(x' + y) =$
   h) $(x + y)(x + y') =$
   i) $(x' + y')(x' + y) =$
   j) $xyz + x'y + xyz' =$
   k) $xy + xy' =$
   l) $(x + y)'(x' + y') =$
   m) $xy + x'z + yz =$
   n) $(x + y')(x + y) =$
   o) $w + (w + wx) =$
   p) $(x' + x')' =$
   q) $(x + x')' =$
   r) $w + (wx'yz) =$
   s) $x'y'z + x'yz + xy' =$
   t) $x'yz + xz =$
   u) $xy + x(wz + wz') =$
   v) $[x(y'z' + yz)]' =$

2. Use DeMorgan's Theorems to simplify the following expressions:
   a) $[(A + D)' \cdot (B' + c)']' =$
   b) $[(A \cdot B \cdot C)' + (C'D)']' =$

3. Given the function *F*. Implement the circuit using AND, OR and inverter gates:

$$F = x'y'z + x'yz + xy'$$

4. Simplify the following Boolean functions to sum-of-products and product-of-sums using three-variable maps. Then draw logic diagrams of the circuits that implement the original and simplified expressions.
   a) $F(x,y,z) = \sum m(0,1,6,7) = \prod M(2,3,4,5)$
   b) $F(x,y,z) = \sum m(0,1,3,4,5) = \prod M(2,6,7)$
   c) $F(x,y,z,w) = \sum m(0,6,8,13,14) + d(x,y,z,w);\ \ d(x,y,z,w) = \sum m(2,4,10)$

5. For the Boolean function
$$F = xy'z + x'y'z + w'xy + wx'y + wxy$$
   a) Obtain the truth table of *F*.
   b) Draw the logic diagram, using the original Boolean expression.
   c) Use Boolean algebra to simplify the function to a minimum number of literals.

6. By means of a timing diagram, show the signal of the output f as a function of the two inputs x_1 and x_2. Use all four possible combinations of x_1 and x_2.